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Visualization of Natural Convection in a Vertical Annular Cylinder with a Partially Heat Source and Varying wall Temperature

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Abstract:
In this work, we visualize the effect of varying wall temperature on the heat transfer by supplying the heat at three different positions to the vertical annular cylinder embedded with porous medium. Finite element method has been used to solve the governing equations. Influence of Aspect ratio(A_r), Radius ratio(R_r) on Nusselt number $(\overline{Nu})$ is presented. The effect of power law exponent effect for different values of Rayleigh number is discussed. The fluid flow and heat transfer is presented in terms of streamlines and isotherms.

Keywords- Natural Convection, Porous medium, varying wall temperature, Aspect Ratio(A_r), Radius ratio(R_r) and Rayleigh number(Ra).

I. Introduction

Convective heat transfer in a porous medium has gained considerable attention of many researchers in recent years. This is justified by the fact that porous media play a vital role in many thermal engineering applications and geophysical applications such as moisture transport in thermal insulations, ceramic processing, the extraction of geothermal energy, nuclear reactor cooling system, underground nuclear waste disposal, energy recovery of petroleum resources, ground water pollution and filtration processes.

In the natural convection aspect the steady natural convection over a semi-infinite vertical wall embedded in a saturated porous medium with wall temperature varying as power function of distance from the origin was discussed by Cheng and Minkowycz [1].

Free convection flow past a vertical flat plate embedded in a saturated porous medium for the case of non-uniform surface temperature was numerically analyzed by Na and Pop [2]. Gorla and Zinalabedini [3] applied the Merk-type series to obtain the local non-similarity solution for the heat transfer from a vertical plate with non-uniform surface temperature and embedded in a saturated porous medium. Seetharamu and Dutta [4] presented free convection in a saturated porous medium adjacent to a non-isothermal vertical impermeable wall. Similarity solution for inclined for mixed boundary region flows in porous media was obtained by Cheng [5]. Both the surface temperatures and free stream velocity must vary according to the same power function of distance from the origin. Later, non-similar solution for mixed convection adjacent to inclined flat surfaces embedded in a porous medium with constant aiding external flow and uniform surface temperature was numerically done by Jang and Ni [6]. Hsieh et al. [7] numerically investigated the non-similarity solutions for mixed convection for vertical surfaces in porous media: variable surface temperature or heat flux. Mixed convection along a non-isothermal vertical plate embedded in the porous medium was studied by Mixed convection along a non-isothermal wedge in a porous medium was numerically considered by Vargas et al. [8], Kumari and Gorla [9].

Saied [10] has considered the case of sinusoidal bottom wall temperature with top cooled and adiabatic vertical walls of a porous medium enclosed in a cavity. The heated wall was assumed to have temperature varying in a sinusoidal wave around a mean temperature greater than that of the top cooled wall. The main finding of this work is the average Nusselt number is increased when the length of the heat source or the amplitude of the temperature variations increased. In another work, Saied [11] investigated similar problem of square porous cavity but boundary conditions were swapped such that the temperature differential was maintained along vertical surfaces. A sinusoidal temperature variation was applied on left vertical surface along with cooled right surface. The bottom and top surfaces are adiabatic.

The effect of non-uniform temperature distribution on an inclined three-dimensional enclosure has been studied by Chao et al [12]. Bottom wall is maintained at a saw –toothed temperature with distribution with different amplitude and orientation, while top wall is isothermal and other faces are adiabatic. The circulation pattern did not change siginificantly with inclination. Cho et al [13] in other study considered half of the Bottom surface cooled, while half of the bottom surface and other vertical surfaces were adiabatic. The results of circulation found to be in good agreement. Fu et al [14] has investigated the natural convection in an enclosure, where the heated wall of the enclosure is divided into two higher and lower temperature and the temperature of the cold wall is maintained at a constant temperature. The results show that the local Nusselt number distribution varies drastically at the intersection of the higher and lower surface temperature regions, and the flow is strongly affected by the above two parameters.

Convection motion in a square cavity with linearly varying temperature imposed along the top surface has been investigated numerically by Shukla, et.al. [15]. The side and bottom walls of the rigid cavity are assumed to be insulated. For low Rayleigh number a single convective cell is formed. With the increase in Rayleigh number, flow and temperature fields became asymmetric. The temperature field is generally stratified with lower art of cavity relatively isothermal. Oosthuizen and Paul [16] considered an enclosure with sidewall partially heated and top wall cooled. In other study, Oosthuizen [17] considered an enclosure with bottom surface heated and the top surface is inclined and maintained at uniform cold temperature. The temperature of the side walls varies in a prescribed way between the bottom and the top wall temperatures. In a study carried out by Sarris et a.l.[18], the top wall is periodically heated while the side walls and the bottom walls are adiabatic. This ensures that the top wall controls the flow of heat. The thermal boundary layer is confined near the top wall. The values of Nusselt number increases with the increase in Rayleigh Number. The work carried out from [19-24] on natural convection in porous medium shown a very good results. A good insight and more applications into the subject are given by Nield and Bejan [27], Vafai[28], Pop and Ingham[29].

The present study focuses on the visualization on the effect of varying wall temperature on the heat transfer behavior in a vertical annular cylinder embedded by porous medium by supplying the heat at three different locations of the vertical annular cylinder. Finite Element Method (FEM) has been used to convert the coupled partial differential equations into a matrix form of equations, which can be solved iteratively with the help of a computer code. The Galerkin Finite Element Method of three nodded triangular elements is used to divide the physical domain of the vertical annular cylinder in to smaller segments, which is a pre-requisite for finite element method. Influence of Aspect ratio(A_r), Radius ratio(R_r), varying wall temperature i.e., power law exponent (λ) and Rayleigh number(Ra) on the average Nusselt number ($\overline{Nu}$) is presented. The effect of power law exponent effect for different values of is discussed. The behavior of fluid flow and heat transfer is presented in terms of streamlines and isotherms.

II. Nomenclature:

A_r Aspect ratio
C_p Specific heat
D_p Particle diameter
g Gravitational accelaration
H_t Height of the vertical annular cylinder
K Permeability of porous media
L Length
P Pressure
$\overline{Nu}$ Average Nusselt number
q_t Total heat flux
r,z Cylindrical co-ordinates
$\bar{r}, \bar{z}$ Non-dimensional co-ordinates
r_i, r_o Inner and outer radius
Ra Rayleigh number

R_r Radius ratio
T Temperature
$\overline{T}$ Non-dimensional Temperature
u Velocity in r direction
w Velocity in z direction
x,y Cartesian co-ordinates
$\bar{x}, \bar{y}$ Non-dimensional co-ordinates

Greek Symbols:

α Thermal diffusity
β_T Coefficient of thermal expansion
ΔT Temperature difference
σ Stephan Boltzmann constant
λ Power law exponent
ρ Density
ν Coefficient of kinematic viscosity
μ Coefficient of dynamic viscosity
φ Porosity
ψ Stream function
$\overline{\psi}$ Non-dimensional Stream function

Subscripts:

w Wall
∞ Conditions at infinity

III. Mathematical Analysis:

A vertical annular cylinder of inner radius r_i and outer radius r_o is considered to investigate the heat transfer behavior. The co-ordinate system is chosen such that the r-axis points towards the width and z-axis towards the height of the cylinder respectively. Because of the annular nature, two important parameters emerge which are Aspect ratio(A_r) and Radius ratio(R_r) of the annulus. They are defined as $A_r = \frac{H_t}{r_o - r_i}, R_r = \frac{r_o - r_i}{r_i}$, where H_t is the height of the annular cylinder. The inner surface of the cylinder is said to be power law function and it varies in the vertical direction along the height of the inner wall of the vertical annular cylinder $T_h = T_\infty + B(z)^\lambda$ and the outer surface is at ambient temperature T_∞. Here λ and B are the constants responsible for temperature variations along the length of the vertical annular cylinder. The top and bottom surfaces of the vertical annular cylinder are adiabatic. It may be noted that, due to axisymmetry only half of the annulus is sufficient for analysis purpose, since other half is mirror image of the first half. The flow inside the porous medium is assumed to obey Darcy law and there is no phase change of fluid. The properties of the fluid and porous medium are homogeneous, isotropic and constant except variation of fluid density with temperature. The fluid and porous medium are in thermal equilibrium. With these assumptions, the governing equations are given by

Continuity Equation: $$\frac{\partial(ru)}{\partial r} + \frac{\partial(rw)}{\partial z} = 0 \quad (1)$$

The velocity in r and z directions can be described by Darcy law as:

Velocity in horizontal direction $u = \frac{-k}{\mu}\frac{\partial p}{\partial r}$

Velocity in vertical direction $v = \frac{-k}{\mu}\left(\frac{\partial p}{\partial z} + \rho g\right)$

The permeability K of porous medium can be expressed as Bejan [28], $k = \frac{D_p^2 \phi^3}{180(1-\phi)^2}$

Momentum Equation: $$\frac{\partial w}{\partial r} - \frac{\partial u}{\partial z} = \frac{gk\beta}{\nu}\frac{\partial T}{\partial r} \quad (2)$$

Energy Equation: $u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\alpha\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right)$ (3)

The continuity equation (1) can be satisfied by introducing the stream function ψ as:

u= $-\frac{1}{r}\frac{\partial\psi}{\partial z}$ (4)

w= $\frac{1}{r}\frac{\partial\psi}{\partial r}$ (5)

The variation of density with respect to temperature can be described by Boussinesq approximation as: $\rho=\rho_\infty[1-\beta_T(T-T_\infty)]$ (6)

The corresponding boundary condition, when heat is supplied at three different locations at the inner wall of the vertical annular cylinder:

At $r=r_i$ and $0\le z\le\frac{H}{6},\frac{5H}{12}\le z\le\frac{7H}{12},\frac{5H}{6}\le z\le H, T_w=T_\infty+B(z)^\lambda,\psi=0$

At $r=r_0$ $T=T_\infty,\psi=0$

The new parameters arising due to cylindrical co-ordinates system are:

Non_dimensional Radius $\bar{r}=\frac{r}{L}$ (7)

Non_dimensional Height $\bar{z}=\frac{z}{L}$ (8)

Non_dimensional Stream function $\bar{\psi}=\frac{\psi}{\alpha L}$ (9)

Non-dimensional Temperature $\bar{T}=\frac{(T-T_\infty)}{(T_w-T_\infty)}$ (10)

Rayleigh Number Ra= $\frac{g\beta_T\Delta TKL}{\nu\alpha}$ (11)

The non-dimensional equations for the heat transfer in vertical cylinder are:

Momentum equation: $\frac{\partial^2\bar{\psi}}{\partial\bar{z}^2}+\bar{r}\left(\frac{1}{\bar{r}}\frac{\partial\bar{\psi}}{\partial\bar{r}}\right)=\bar{r}Ra\frac{\partial\bar{T}}{\partial\bar{r}}$ (12)

Energy Equation: $\frac{1}{\bar{r}}\left[\frac{\partial\bar{\psi}}{\partial\bar{r}}\frac{\partial\bar{T}}{\partial\bar{z}}-\frac{\partial\bar{\psi}}{\partial\bar{z}}\frac{\partial\bar{T}}{\partial\bar{r}}\right]=\left(\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}\left(\bar{r}\frac{\partial\bar{T}}{\partial\bar{r}}\right)+\frac{\partial^2\bar{T}}{\partial\bar{z}^2}\right)$ (13)

The corresponding non-dimensional boundary conditions, when heat is supplied at three different locations at the inner wall of the vertical annular cylinder

At $r=r_i$ and $0\le z\le\frac{H}{6},\frac{5H}{12}\le z\le\frac{7H}{12},\frac{5H}{6}\le z\le H,\bar{T}=T_\infty+B(z)^\lambda,\bar{\psi}=0$

At $r=r_0$ $\bar{T}=0,\ \bar{\psi}=0$

IV. Method of Solution:

Equations (12) and (13) are the coupled partial differential equations to be solved in order to predict the heat transfer behavior. These equations are solved by using finite element method. Galerkin approach is used to convert the partial differential equations into a matrix form of equations. A simple 3-noded triangular element is considered. The polynomial function for Tcan be expressed as $T=\alpha_1+\alpha_2 r+\alpha_3 z$ (14)

The variable T has the value $T_i, T_j\ and\ T_k$ at the nodal positions i,j,k of the element. The r and z co-ordinates at these points are r_i, r_j, r_k and z_i, z_j, z_k respectively. Since $T=N_iT_i+N_jT_j+N_kT_k$

Where N_i, N_j & N_k are shape functions given by

$N_m = \frac{a_m + b_m r + c_m z}{2A}$, $m = 1, 2, 3$ Where a_m, b_m, c_m are matrix coefficients.

Following the same procedure the Eq(12) becomes:

$$\frac{2\pi\overline{R}}{4A}\left[\begin{bmatrix} b_1^2 & b_1b_2 & b_1b_3 \\ b_1b_2 & b_2^2 & b_2b_3 \\ b_1b_3 & b_2b_3 & b_3^2 \end{bmatrix} + \begin{bmatrix} c_1^2 & c_1c_2 & c_1c_3 \\ c_1c_2 & c_2^2 & c_2c_3 \\ c_1c_3 & c_2c_3 & c_3^2 \end{bmatrix}\right]\begin{Bmatrix} \overline{\psi}_1 \\ \overline{\psi}_2 \\ \overline{\psi}_3 \end{Bmatrix}$$

$$=- \frac{2\pi\overline{R}^2 Ra}{6}\begin{Bmatrix} b_1\overline{T}_1 + b_2\overline{T}_2 + b_3\overline{T}_3 \\ b_1\overline{T}_1 + b_2\overline{T}_2 + b_3\overline{T}_3 \\ b_1\overline{T}_1 + b_2\overline{T}_2 + b_3\overline{T}_3 \end{Bmatrix}$$

The stiffness matrix of Energy Equation is:

$$\left[\frac{2\pi}{12A}\begin{Bmatrix} c_1\overline{\psi}_1 + c_2\overline{\psi}_2 + c_3\overline{\psi}_3 \\ c_1\overline{\psi}_1 + c_2\overline{\psi}_2 + c_3\overline{\psi}_3 \\ c_1\overline{\psi}_1 + c_2\overline{\psi}_2 + c_3\overline{\psi}_3 \end{Bmatrix}\begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix} - \frac{2\pi}{12A}\begin{Bmatrix} b_1\overline{\psi}_1 + b_2\overline{\psi}_2 + b_3\overline{\psi}_3 \\ b_1\overline{\psi} + b_2\overline{\psi}_2 + b_3\overline{\psi}_3 \\ b_1\overline{\psi}_1 + b_2\overline{\psi}_2 + b_3\overline{\psi}_3 \end{Bmatrix}\begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix}\right]\begin{Bmatrix} \overline{T}_1 \\ \overline{T}_2 \\ \overline{T}_{31} \end{Bmatrix}$$

$$+\frac{2\pi\overline{R}}{4A}\left\{\begin{bmatrix} b_1^2 & b_1b_2 & b_1b_3 \\ b_1b_2 & b_2^2 & b_2b_3 \\ b_1b_3 & b_2b_3 & b_3^2 \end{bmatrix}\begin{Bmatrix} \overline{T}_1 \\ \overline{T}_2 \\ \overline{T}_3 \end{Bmatrix} + \begin{bmatrix} c_1^2 & c_1c_2 & c_1c_3 \\ c_1c_2 & c_2^2 & c_2c_3 \\ c_1c_3 & c_2c_3 & c_3^2 \end{bmatrix}\begin{Bmatrix} \overline{T}_1 \\ \overline{T}_2 \\ \overline{T}_3 \end{Bmatrix}\right\} = 0 \qquad (15)$$

V. Results and Discussion:

Results are obtained in terms of Nusselt number at hot wall for various parameters such as power law exponent $'\lambda'$, Aspect Ratio(A_r), Radius ratio(R_r) and Rayleigh number(Ra), when heat is supplied at three different locations of the hot wall of the vertical annular cylinder.

The average Nusselt number is given by $\overline{Nu} = -\frac{1}{L}\int_0^L \left(\frac{\partial\overline{T}}{\partial\overline{r}}\right)_{\overline{r}=r_i,r_0}$ (16)

Where L is the length of the heated wall of the vertical annular cylinder. $i.e., L = L_1 + L_2 + L_3$

a)

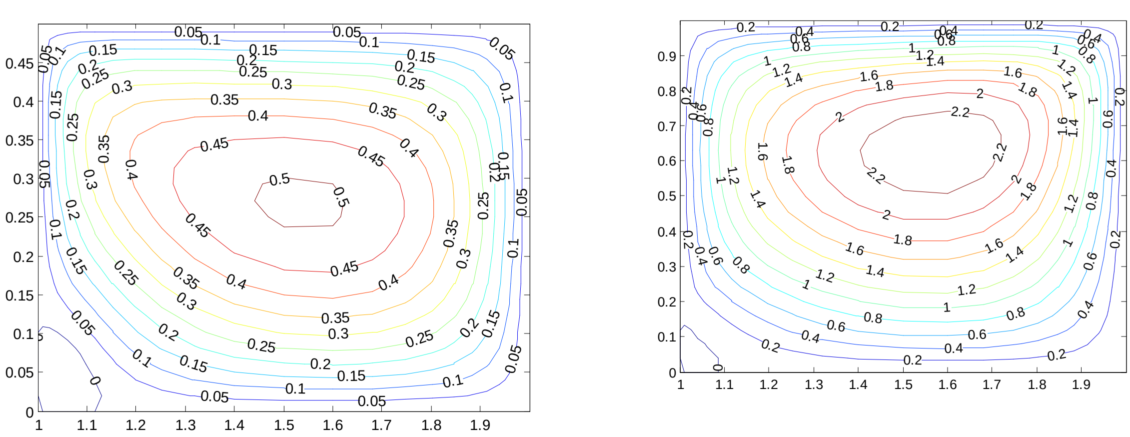
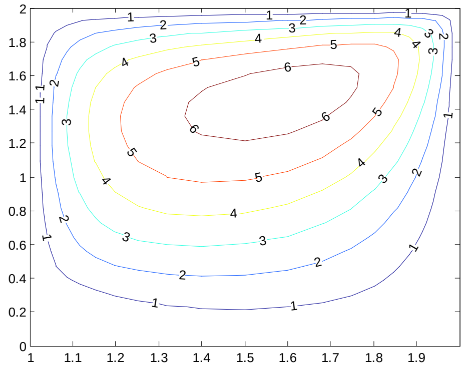
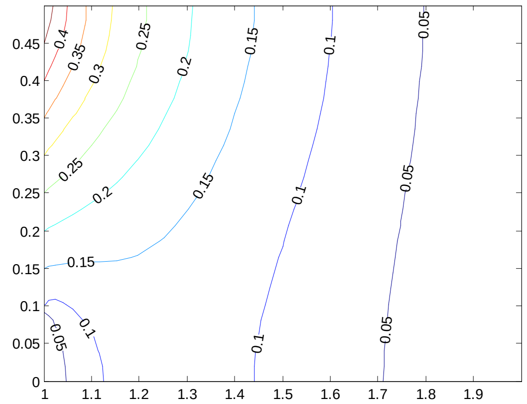


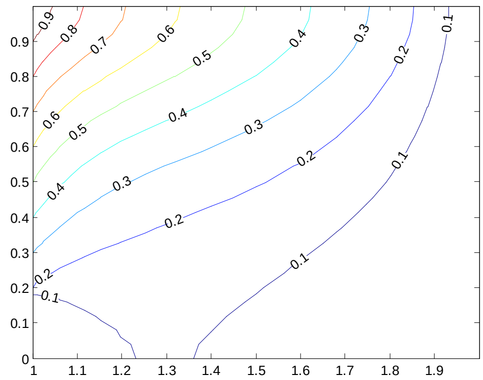
Fig:1: Streamlines(left) and Isotherms(Right) for λ =1,R_r =1, Ra=50
a) A_r =0.5 b) A_r =1 c) A_r =2

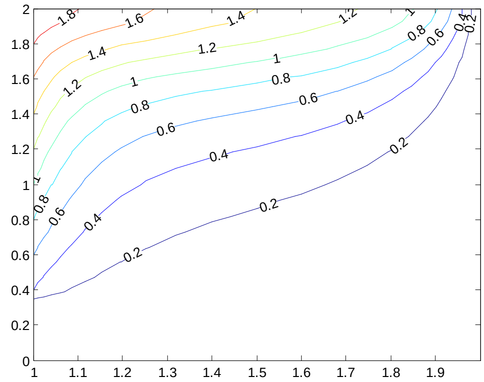
b)


0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9

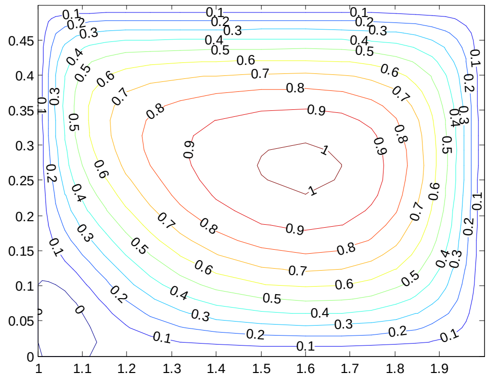

0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45
1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9

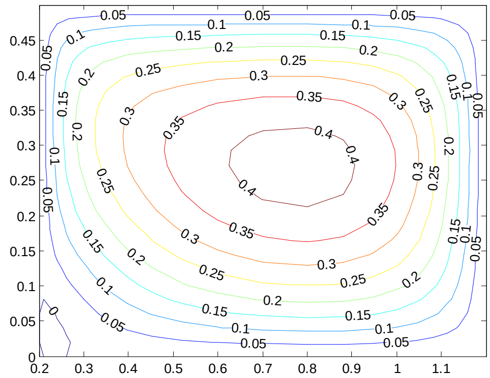
c)


0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9

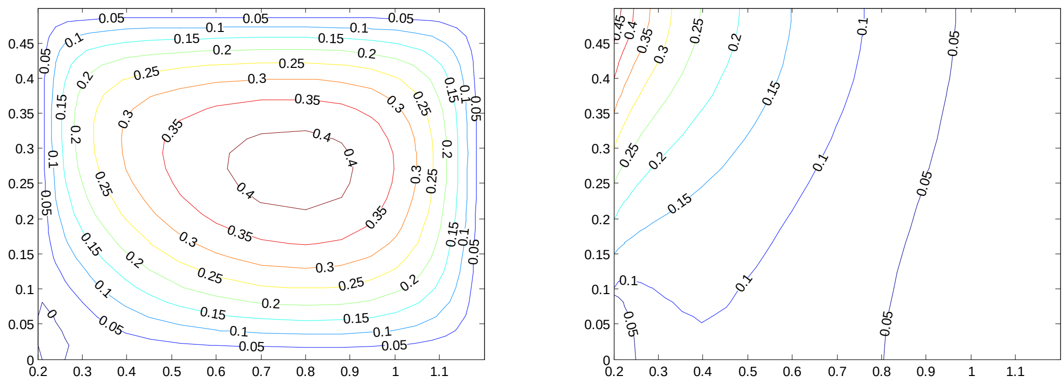

0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9

a)


0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45
1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9


0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1

b)



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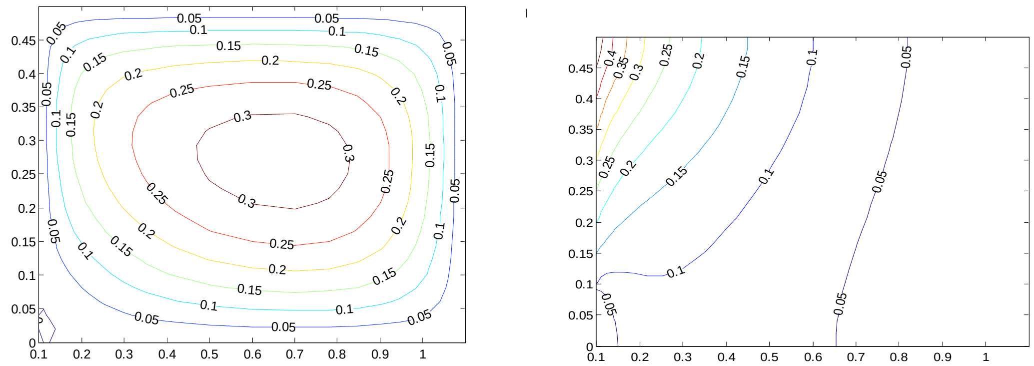
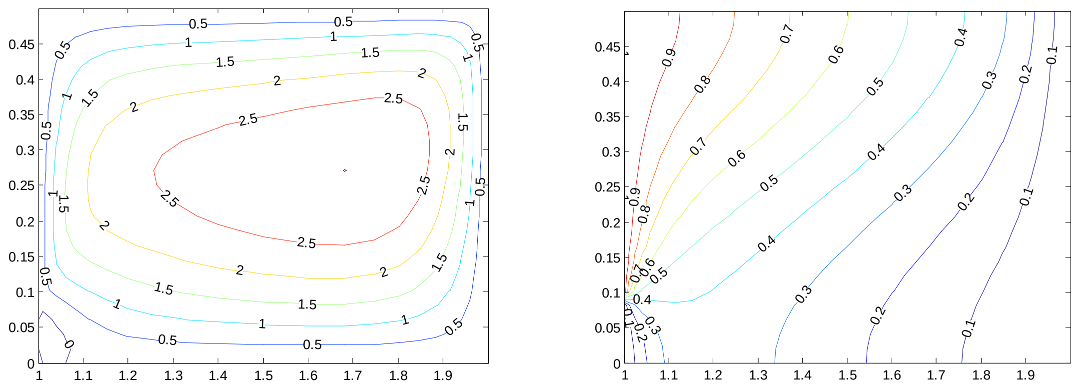


Fig:2: Streamlines(left) and Isotherms(Right) for λ =1 , A_r =0.5, Ra=100
a) R_r =1 b) R_r =5 c) R_r =10

a)



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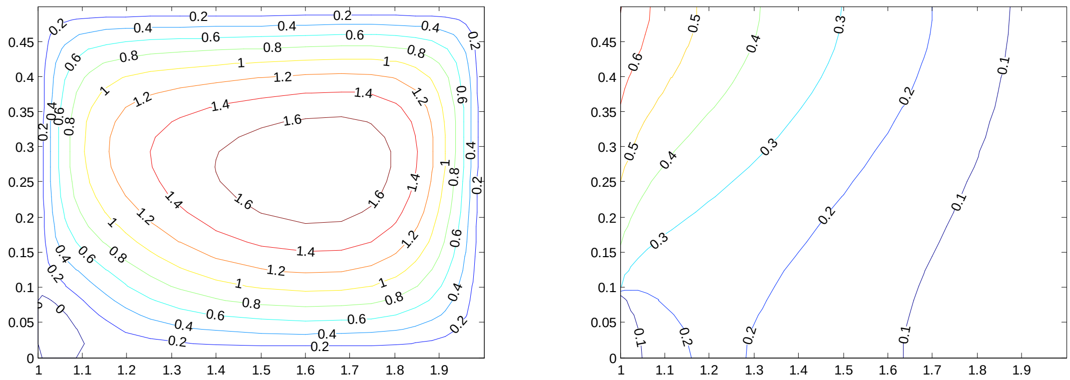

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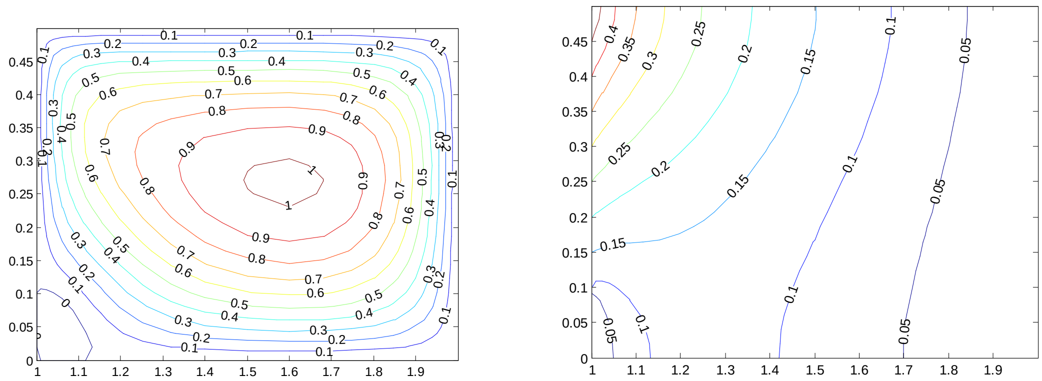
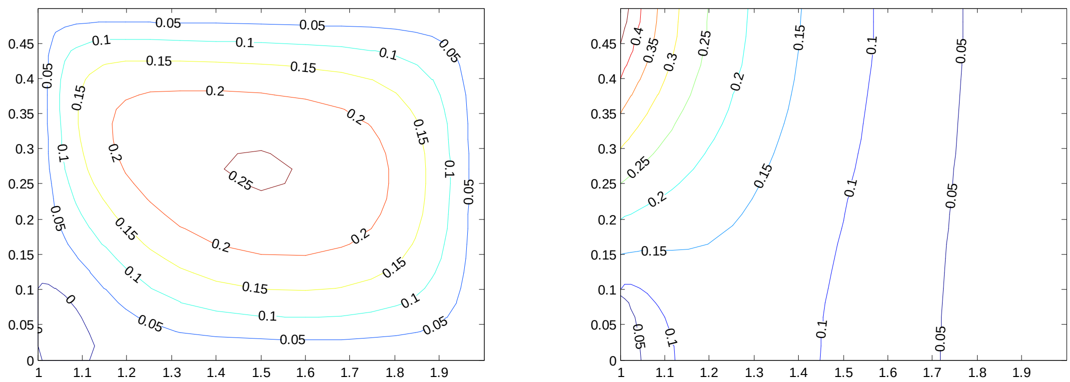


Fig:3: Streamlines(left) and Isotherms(Right) for A_r =0.5, R_r =1, Ra=100
a) λ=0 b) λ =0.5 c) λ=1

a)



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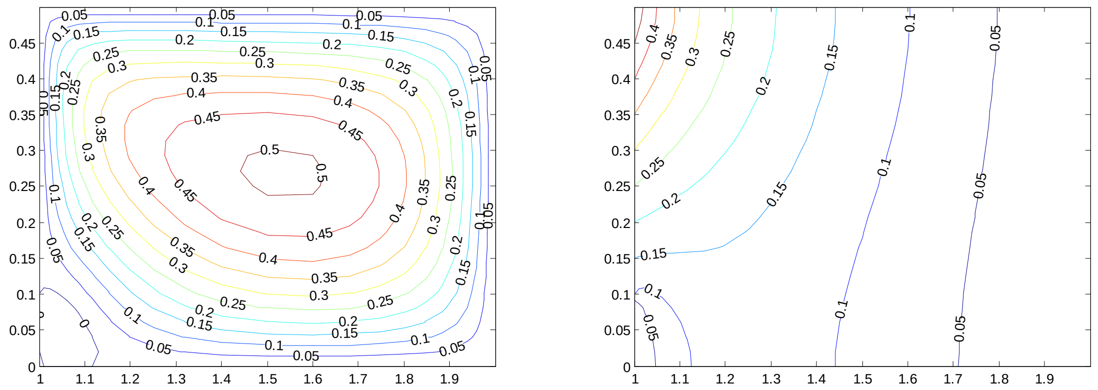

c)

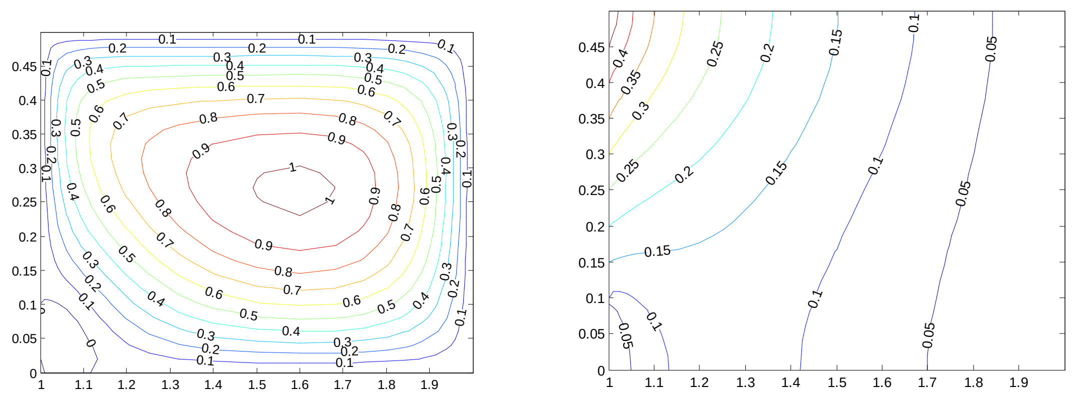

Fig:4: Streamlines (left) and Isotherms(Right) for A_r =0.5, R_r =1, λ=1
a)Ra=25 b) Ra =50 c) Ra =100

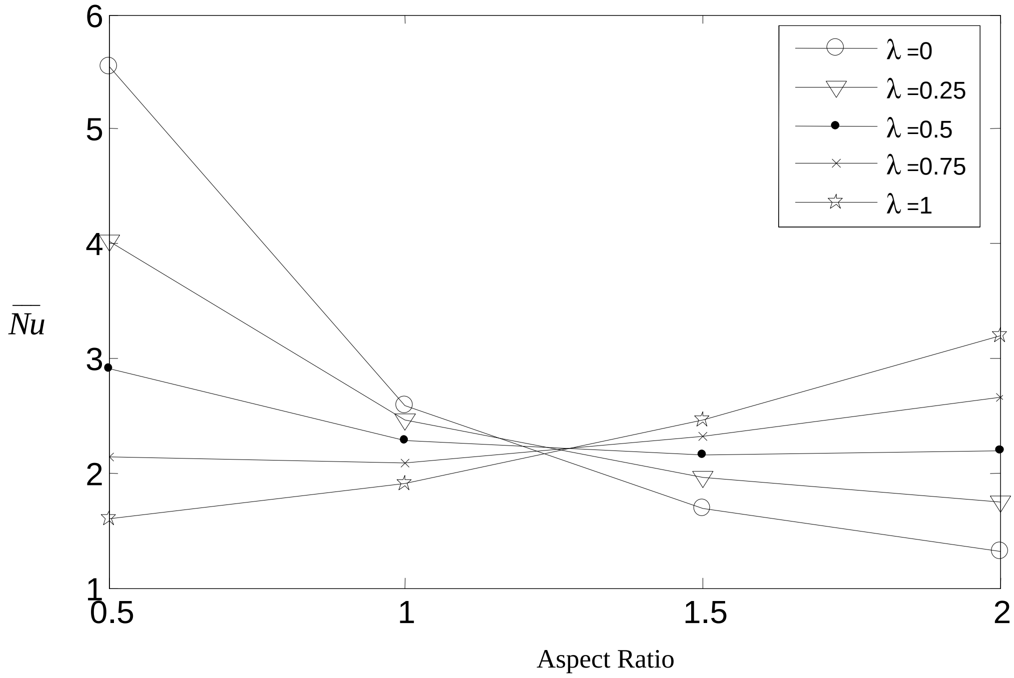

Fig: 5: $\overline{Nu}$ Variations with A_r at **hot wall** for different values of λ at R_r=1, Ra=50

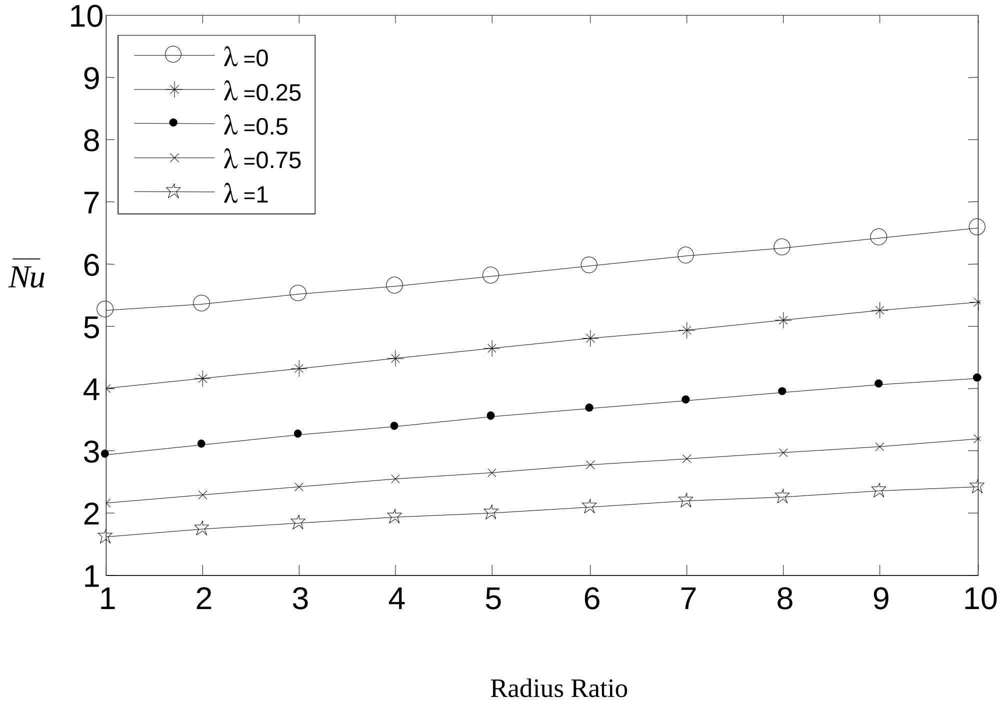

Fig: 6: $\overline{Nu}$ Variations with R_r at **hot wall** for different values of λ at A_r =0.5, Ra=100

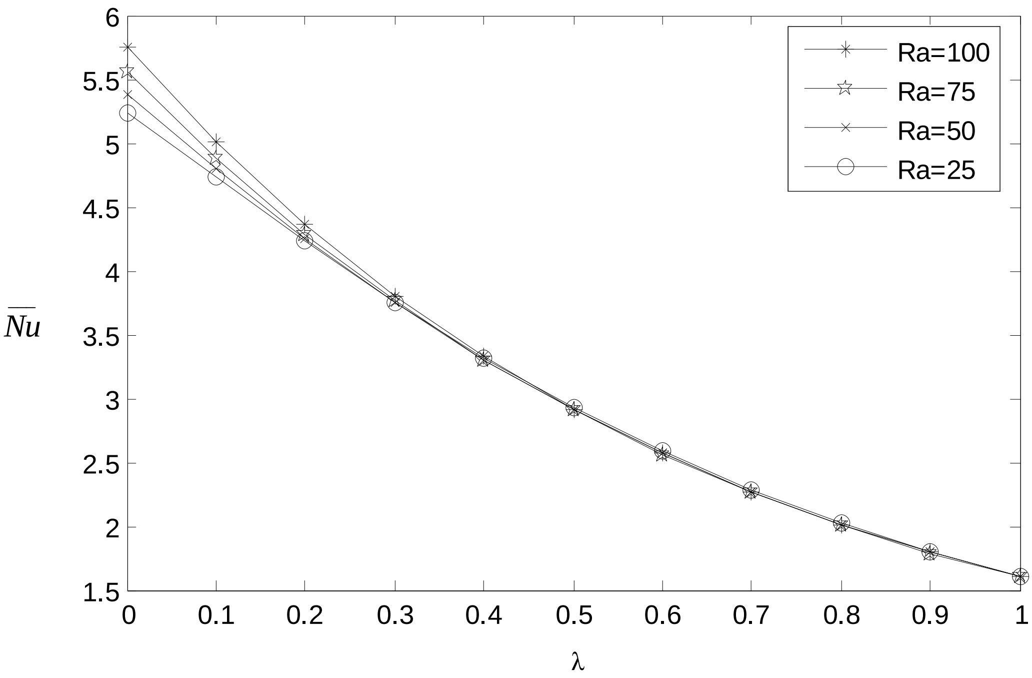

Fig: 7: $\overline{Nu}$ Variations with λ at **hot wall** for different values of Ra at A_r=0.5, R_r=1

Fig.1 shows the evolution of streamlines and isothermal lines inside the porous medium for various values of Aspect Ratio(A_r) for$\lambda = 1, R_r = 1, Ra = 50$. It is clear from the streamlines and isothermal lines that the thermal boundary layer thickness decreases as the Aspect ratio(A_r) increases. At low Aspect ratio(A_r) the streamlines tend to occupy the whole domain of the vertical annular cylinder as compared to the higher values of Aspect ratio(A_r). It is clearly seen that more convection heat transfer takes place at the upper portion of the vertical annular cylinder. The streamlines and isothermal lines shift from the left upper portion of the cold wall of vertical annular cylinder as the Aspect ratio(A_r) increases.

Fig.2 illustrates the distribution of streamlines and isothermal lines inside the porous medium of vertical annular cylinder for various values of Radius Ratio(R_r), corresponding to $\lambda = 1, A_r = 0.5, Ra = 100$. The streamlines and isothermal lines tends to move towards the hot wall and move away from the cold wall of the vertical annular cylinder as Radius Ratio(R_r) increases. Which in turn, increases the thermal gradient at hot wall and decreases the same at cold wall of the vertical annular cylinder. This alludes that the heat transfer rate increases at the hot wall and decreases at cold wall with increase in Radius Ratio(R_r). Thus the thermal boundary layer becomes thinner with increase in Radius Ratio(R_r).

Fig.3 depicts the evolution of streamlines and isothermal lines inside the porous medium for various values of power law exponent (λ) for$A_r = 0.5$, $R_r = 1$, $Ra = 100$. The fluid gets heated up near the hot wall and moves up towards the cold wall due to high buoyancy force and then returns back to hot wall of the vertical annular cylinder. For the case of isothermal wall temperature $(\lambda = 0)$, the magnitude of the streamlines is high as compared to the non-isothermal temperature $(\lambda \neq 0)$. It can be seen from streamlines and isothermal lines that the fluid movement shifts towards the cold wall of the vertical annular cylinder, when the value of power law exponent 'λ' is varied from 0 to 1. Thus the circulation of the fluid decreases with the increase in power law exponent (λ).

Fig.4 shows the evolution of streamlines and isothermal lines inside the porous medium for various values of Rayleigh number (Ra) for $A_r = 0.5$, $R_r = 1$, $\lambda = 1$. The increases Rayleigh number(Ra) promotes the fluid movement due to higher buoyancy force, which in turn allows the convection heat transfer to be dominant. So the magnitude of the streamlines increases with the increase in Rayleigh number (Ra).

Fig.5 illustrates the average Nusselt number $(\overline{Nu})$ at hot wall with respect to Aspect ratio(A_r) for various values of power law exponent(λ), corresponding to $R_r = 1$, $Ra = 50$. For a given value of Aspect ratio(A_r), the average Nusselt number $(\overline{Nu})$ decreases with the increase in power law exponent(λ). This happens due to the reason that the heat content of the wall is more at wall temperature $(\lambda = 0)$ as compared to the non-isothermal temperature ($\lambda \neq 0$), which leads to increased fluid movement at the hot wall, which in turn increases the average Nusselt number ($\overline{Nu}$). Whereas, it is found that the average Nusselt number ($\overline{Nu}$) decreases by 71%, when the power law exponent(λ) increases from 0 to 1. So, there is a decrease in average Nusselt number ($\overline{Nu}$) for higher values of Aspect ratio(A_r).

Fig.6 visualize, the average Nusselt number ($\overline{Nu}$) at hot wall of the vertical annular cylinder, with respect to Radius Ratio(R_r) for various values of power law exponent(λ), corresponding to $A_r = 0.5,\ Ra = 100$. It is found that the average Nusselt number ($\overline{Nu}$) increases with increase in Radius Ratio(R_r). But, it is seen that the average Nusselt number ($\overline{Nu}$), decreases with increase in power law exponent(λ). So, for a given Radius Ratio(R_r), the difference between the average Nusselt number ($\overline{Nu}$) at two different values of power law exponent(λ) increases with increase in power law exponent(λ). For instance, at $R_r = 1$, the average Nusselt number ($\overline{Nu}$), decreased by 69.32%, when power law exponent(λ) increased from 0 to 1.

Fig.7 shows the variation of the average Nusselt number ($\overline{Nu}$) at hot wall of the vertical annular cylinder, with respect to power law exponent(λ), for various values of Rayleigh number (Ra) corresponding to $A_r = 0.5, R_r = 1$. It is found that the average Nusselt number ($\overline{Nu}$), decreases with the increase in power law exponent(λ). It can be seen that the average Nusselt number ($\overline{Nu}$) increases with increase in Rayleigh number (Ra) for small values of power law exponent(λ). As the value of power law exponent(λ) increases beyond 0.5, practically there is no effect of Rayleigh number (Ra) on the average Nusselt number ($\overline{Nu}$). At the isothermal wall temperature($\lambda = 0$), the average Nusselt number ($\overline{Nu}$), decreased by 9% , when Rayleigh number (Ra) is increased from 25 to 100. The corresponding decrease in the average Nusselt number ($\overline{Nu}$) at $\lambda = 0.5$, is found to be 0.37%. This shows that the difference between the average Nusselt number ($\overline{Nu}$) at two different values of Rayleigh number (Ra) decreases for non-isothermal temperature ($\lambda \neq 0$) as compared to that of isothermal temperature ($\lambda = 0$).

VI. Conclusion:

It is found that more convection heat transfer takes place at the three positions of heated wall of the vertical annular cylinder at higher values of varying wall temperature $'\lambda'$ and Rayleigh number(Ra). The magnitude of streamlines decreases with increase in Radius ratio (R_r). The effect of Rayleigh number on the average Nusselt number ($\overline{Nu}$) becomes insignificant with the increase of varying wall temperature, i.e., power law exponent $'\lambda'$.

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References:

[1.] P. Cheng and W. J. Minkowycz, "*Free convection about a vertical flat plate embedded in a porous medium with application to heat transfer from a dike*," Journal of Geophysical Research, vol. 82, No. 14, pp. 2040–2044, (1977).
[2.] T.Y. Na and I. Pop, "*Free convection flow past a vertical flat plate embedded in a saturated porous medium*," International Journal of Engineering Science, vol. 21, No. 5, pp. 517–526, (1983).
[3.] R. S. R. Gorla and A. H. Zinalabedini, "*Free convection from a vertical plate with non-uniform surface temperature and embedded in a porous medium*," Journal of Energy Resources Technology, vol. 109, No. 1, pp. 26–30, (1987).
[4.] K.N Seetharamu and P. Dutta, "*Free Convection in a Saturated Porous Medium adjacent to a Non-isothermal Vertical Impermeable Wall*", *Heat and Mass Transfer*, Vol. 25, No.1, pp. 9-15, (1990).
[5.] Ping Cheng. "*Combined Free and Convection Flow about Inclined surfaces in porous media*", International Journal of Heat and Mass Transfer, Vol. 20, Issue 8, pp.807-814, (1977).
[6.] J. Y. Jang and J. R. Ni, "*Mixed convection adjacent to inclined flat surfaces embedded in a porous medium*", Warme-und Stoffubertragung, Vol.27, pp.103-108, (1992).
[7.] J. C. Hsieh, T. S. Chen, and B. F. Armaly, "*Non similarity solutions for mixed convection from vertical surfaces in porous media: variable surface temperature or heat flux*," International Journal of Heat and Mass Transfer, vol. 36, No. 6, pp. 1485–1493, (1993).
[8.] J. V. C. Vargas, T. A. Laursen and A. Bejan, "*Non-similar solutions for mixed convection on a wedge embeded in a porous medium*", International Journal of Heat and Fluid Flow, Vol.16, pp.211–216, (1995).
[9.] M. Kumari and R. S. R. Gorla, "*Combined convection along a non-isothermal wedge in*

a porous medium," Heat and Mass Transfer, vol. 32, No. 5, pp. 393–398, (1997).

[10.] N.H. Saeid, "*Natural Convection in porous cavity with sinusoidal bottom wall temperature variation*", International Communications in Heat and Mass Transfer, Vol.32, pp.454-463, (2005).

[11.] N.H. Saeid and A.A. Mohammed, "*Periodic free convection from a vertical plate in a saturated porous medium, non-equilibrium model*', International Journal of Heat and Mass Transfer, Vol.48, pp.3855-3863, (2005)

[12.] P. Chao, H.Ozoe, S.W. Churchill, "*Effect of a non-uniform surface temperature on a laminar natural convection in a rectangular enclosure*", Chemical Engineering Communications, Vol.9, pp.245-254, (1981).

[13.] PK.B.Chao, H.Ozoe, S.W.Churchill, N.Lior, "*Laminar natural convection in an inclined rectangular box with the lower surface half-heated and half-insulated*", Journal of Heat and Mass Trasnfer, Vol.105, pp.425-432, (1983).

[14.] W.S.Fu, C.C.Tseng, Y.C.Chen, "*Natural Convection in an enclosure with non-uniform wall temperature*", International Communications in Heat and Mass Transfer, Vol.21, pp.819-828, (1994).

[15.] V.Shukla, R.Murtigudde, V.Prasad, M.Cane, "*Natural Convection in a horizontal cavity with a linear temperature variation on the top*", 27th National Heat Transfer Conference, Mixed Convection Heat Transfer, ASME HTD, Vol.163, pp.1-8, (1991).

[16.] P.H.Oosthuizen, "*Free Convection in a square cavity with a partially heated wall and cooled top*", Journal of Thermophysics Heat Transfer, Vol.5, pp.583-588, (1991).

[17.] P.H. Oosthuizen, "*Free convection flow in an enclosure with a cooled inclined upper surface*", Computational Mechanics, Vol.14, pp.420-430, (1994).

[18.] I.E. Sarris, I.Lekakis, N.S. Vlachos, "*Natural Convection in a 2D enclosure with sinusoidal upper wall temperature", Numerical Heat Transfer*, Part A 42, pp.513-530, (2002).

[19.] A. Bejan and K. R. Khair, "*Heat and mass transfer by natural convection in a porous medium,"*International Journal of Heat and Mass Transfer, vol. 28, No. 5, pp. 909–918, (1985).

[20.] F. C. Lai and F. A. Kulacki, "*Coupled heat and mass transfer by natural convection from vertical surfaces in porous media*," International Journal of Heat and Mass Transfer, vol. 34, No. 4-5, pp. 1189–1194, (1991).

[21.] A. Nakayama and M. A. Hossain, "*An integral treatment for combined heat and mass transfer by natural convection in a porous medium*," International Journal of Heat and Mass Transfer, vol. 38, No. 4, pp. 761–765, (1995).

[22.] Jang, J.Y., and W.J. Chang, "*Buoyancy-induced inclined boundary layer flow in a saturated porous medium resulting from combined heat and mass buoyancy effects*", International Communication Heat and Mass Transfer Vol.15, pp.17–30, (1988).

[23.] P. Singh and Queeny, "*Free convection heat and mass transfer along a vertical surface in a porous medium,*" Acta Mechanica, vol. 123, No. 1–4, pp. 69–73, (1997).

[24.] F. C. Lai and F. A. Kulacki, "*Influence of lateral mass flux on mixed convection over inclined surfaces in saturated porous media,*" International Journal of Heat and Mass Transfer, vol. 112, No. 2, pp. 515–518, (1990).

[25.] A. Bejan "*Convective Heat Transfer*", 2nd edition, New York, John Wiley & Sons, (1995).

[26.] A. E. Bergles, *Handbook of Heat Transfer, McGraw-Hill*, New York, NY, USA, 3rd edition, (1998).

[27.] Nield, D.A., *A. Bejan, A:Convection in Porous Media*, Second edition, Springer, New York, (1999).

[28.] Vafai, K (Ed.): *Hand book of Porous media*, Marcel Dekker, New York, (2000).

[29.] I. Pop and D. B. Ingham, *Convective Heat Transfer: Mathematical and Computational Modeling of Viscous Fluids and Porous Media*, Pergamon, Oxford, UK, (2001).

[30.] M.Elshayeb and Y.K.Beng, "*Applications of finite difference and finite element methods for thermal problems*", University Malaysia Sabah, Sabah, (2000).

[31.] R.W.Lewis, P.Nithiarasu and K.N. Seetharamu,, "*Fundamentals of the finite element method for heat and fluid flow*", John, Wiley and Sons, Chichester, (2004).

[32.] L. J. Segerland, "*Appliled Finite Element Analysis*", John Wiley and Sons, New York, (1982).